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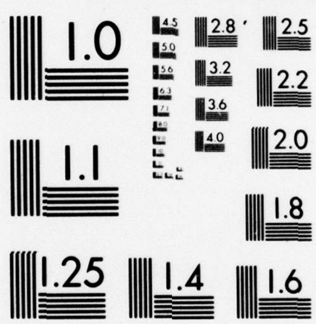
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DYNAMIC STRUCTURAL ANALYSIS WITH SUBSTRUCTURES

by

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and
D.T. Nguyen, Graduate Research Assistant

December 1978

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A method for dynamic structural analysis with substructures and the subspace iteration is developed. The method does not use component mode synthesis concepts. Therefore eigenproblem for each substructure is not solved. The method uses only substructural stiffness matrices and the mass matrix for each finite element of the system. The mass matrix for the entire structure or any of its substructures need not be computed. However, efficiency of the method is improved when mass matrix for the entire structure is computed and saved in the computer core. (cont'd.)

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Unlike component mode substitution methods, no approximating assumptions are made. Thus, natural frequencies and mode shapes for the finite element model employed are the same with or without the substructuring algorithm. This is demonstrated by computing first ten natural frequencies and the corresponding mode shapes for an open truss helicopter tail-boom structure.

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I. INTRODUCTION

Analysis of structures for transient dynamic loads is of considerable importance in many fields of engineering. Design of aircraft structures, spacecraft structures, ship structures, machine components and framed structures for earthquake loading are some examples where transient dynamic analysis procedures are required. Many of these structures must be represented by large finite element models. Therefore it is desirable to divide the structure into smaller components or substructures for efficient dynamic analysis.

This paper addresses the question of dynamic analysis of large structures with substructures or components. A simple and efficient numerical method for such analysis is developed. The method does not require generation and storage of stiffness and mass matrices for the entire structure. It requires generation of only substructural stiffness matrices and storage of their decomposed factors. Thus the method is quite compact requiring minimum computer storage.

The method of modal superposition is adopted for dynamic structural analysis. In this approach free vibration modes and the corresponding natural frequencies must be first computed. This information is then used to define uncoupled equations of motion for the system. Uncoupled equations of motion are then easily solved using Duhamel Integral. Thus the problem of structural dynamics is reduced to finding natural frequencies and mode shapes with substructures.

Methods of component mode substitution for calculating system eigenvalues and corresponding eigenvectors are briefly reviewed in the

next subsection. Section II defines the dynamic structural analysis problem. the method of dynamic structural analysis with substructures is developed in Section III. An example problem is considered in Section IV to demonstrate applicability of the method.

Component Mode Substitution Methods

In component mode substitution methods of free vibration analysis, the structure is divided into a number of smaller structures. Mode shapes of each substructure are calculated for a variety of boundary conditions. Compatibility and equilibrium conditions at the inferfaces of all substructures are invoked and a reduced eigenvalue problem for the entire structure is generated. After the reduced eigenvalue problem has been solved, system mode shapes are generated using previously defined transformations in terms of component modes.

Advantage of component mode substitution methods is in the experimental determination of system mode shapes. It is easier to measure mode shapes and natural frequencies of smaller components as compared to measuring mode shapes for the entire system. Knowing mode shapes of each component, a method is needed to synthesize the system modes. The component mode substitution methods [1-7] then provide such a means for generating system modes.

Many variations of the component mode substitution method have been presented since Hurty's 1960 paper on the subject [1]. Several review articles on the subject have appeared in the literature [2,3,4,5]. Some

other recent articles that deal with component mode substitution concept are the Refs. 6 and 7.

The method developed in the paper does not use component mode substitution concept. Therefore details of these techniques are not presented here. Reader may consult Refs. 1-7 and articles cited in them for more details on component mode substitution methods. In the method presented in the paper one calculates eigenmodes and the corresponding eigenvalues for the system with the substructural data, but without calculating mode shapes of any substructure. No assumptions other than the usual assumptions of linear elastic systems modeled by finite elements, are made.

II. PROBLEM OF STRUCTURAL DYNAMICS

The finite element technique of analysis is used. The equation of motion for a typical finite element model for the structural system with n independent degrees-of-freedom can be written as

$$M\ddot{u} + C\dot{u} + Ku = F(t) \tag{1}$$

with initial conditions specified on u and $\dot{u}$. Here M, C and K are (n x n) mass, damping and stiffness matrices, respectively; u, $\dot{u}$ and $\ddot{u}$ are the displacement, velocity and acceleration vectors, respectively and F(t) is a forcing function. These equations are derived using the Lagrangian approach [8]. If it is assumed that the system described by Eq. (1) is linear, then M, C and K are constant; that is they are not functions of time or motion.

Most commonly used procedure of dynamic analysis of linear systems is the normal modes method. This is also referred to as the modal or mode superposition analysis and is adopted in the present paper. In this method, one first considers the free vibration problem:

$$M\ddot{u} + Ku = 0 \tag{2}$$

Substituting

$$u = \phi \sin\omega(t - t_o) \tag{3}$$

into Eq. (2), the following generalized eigenvalue problem is obtained:

$$K\phi = \omega^2 M\phi \tag{4}$$

The n eigenvalues of Eq. (4) give the natural frequencies of the system and the corresponding eigenvectors are the mode shapes. The complete solution to Eq. (4) can be written as

$$K\Phi = M\Phi\Omega^2 \tag{5}$$

in which the columns in Φ are the eigenvectors ϕ^i and $\Omega^2 = \text{diag}\ (\omega_i^2)$.

The eigenvectors of Eq. (4) are linearly independent and can be orthonormalized with respect to the mass matrix M. These vectors, therefore, form a basis for the n dimensional vector space. The vector u(t) can be expressed as a linear combination of the eigenvectors ϕ^i. This relationship may be expressed as the following linear transformation

$$u(t) = \Phi\eta(t) \tag{6}$$

where $\eta(t) \in R^n$ is usually called the normal coordinate vector. Substituting Eq. (6) into Eq. (1) and pre-multipying by Φ^T, then givs

$$\ddot{\eta} + C^*\dot{\eta} + \Omega^2\eta = F^*(t) \tag{7}$$

Here $F^*(t) = \Phi^T F(t)$, $\Phi^T K\Phi = \Omega^2$ and $C^* = \Phi^T C\Phi$. If the damping matrix C for the system is of a restricted form [9,10], then C* is a diagonal matrix. Equation (7) then represents a system of n uncoupled equations. These equations can be solved exactly using the Duhamel Integral [8,10].

It is noted that the finite element model for practical systems is generally quite large; that is n is generally very large. For many dynamic analysis problems, it is reasonable to assume that only a first few eigenmodes contribute significantly to the dynamic response. Thus, approximate analyses are carried out with only p << n eigenvectors. In

this case, Φ forms an M-orthonormal basis for the p dimensional vector space and Eq. (7) represents a system of only p uncoupled equations. Integration of Eq. (7) determines the normal coordinates $\eta \in R^p$, and Eq. (6) then determines the real displacement u.

III. DYNAMIC ANALYSIS WITH SUBSTRUCTURES

The problem of dynamic structural analysis is reduced to formulation of the eigenvalue problem of Eq. (4) and generation of the desired eigenvectors and eigenvalues. Once eigenvalues and eigenvectors are known, the uncoupled system of Eqs. (7) is readily obtained.

Several methods for solving the eigenvalue problem of Eq. (4) have been developed in the literature [11-15]. These methods take advantage of bandedness property of the K and M matrices. It has been recognized that it is desirable to divide a large structure into a number of smaller structures for the purpose of calculating system eigenmodes and eigenvalues. The stiffness and mass properties of a substructure are used to calculate its eigenmodes. This requires solution of several smaller eigenvalue problems. Eigenmodes of all substructures are then synthesized to generate system eigenmodes. Different methods of synthesis generate various component mode substitution methods [5]. Most of these methods generate approximate eigenvalues and eigenvectors for the system.

The method presented in the paper, however, does not use the component mode synthesis concept, since the eigenmodes for substructures are never computed. The method directly generates the desired number of system eigenmodes and eigenvalues using the substructural stiffness matrices and the mass matrix for each finite element. The stiffness or mass matrix for the entire system is never generated. Moreover the mass matrix for any substructure is not required to be generated and stored.

The method developed here uses the subspace iteration algorithm of

Ref. 12 for solving the eigenvalue problem of Eq. (5). The basic idea of the subspace iteration method is quite similar to the general Rayleigh-Ritz method. In the method, one starts with a set of $p \ll n$ linearly independent vectors. These vectors form a basis for the p-dimensional vector space. The mode shapes for the system are expressed as linear combinations of the assumed basis vectors. The parameters of these linear combinations are obtained by solving a reduced eigenvalue problem. An iterative process is then used to improve the assumed basis vectors until convergence is obtained.

It is suggested [12] that when p mode shapes are required, one should start with $q > p$ linearly indpendent vectors, where q is given as

$$q = \min\{2p,\ p+8,\ n\} \tag{8}$$

This improves accuracy of the first p eigenvectors and the corresponding eigenvalues. Let us define a linear transformation

$$\Phi^{(0)\dagger} = X^{(0)}\Phi^* \tag{9}$$

where $X^{(0)}$ is an (n x q) matrix whose columns are the assumed basis vectors for R^q and Φ^* is a (q x q) matrix whose elements are the unknown multipliers of the transformation. Substituting Eq. (9) into Eq. (5) and premultiplying by $X^{(0)^T}$ one obtains

$$K^*\Phi^* = M^*\Phi^*\Omega^{*2} \tag{10}$$

where

$$K^* = X^{(0)^T} K X^{(0)} \quad , \qquad M^* = X^{(0)^T} M X^{(0)} \tag{11}$$

---

† The superscript indicates the iteration number.

Equation (10) is a reduced eigenproblem of dimension q that is solved for Φ^* and Ω^{*2}. Equation (9) is then used to obtain approximate system mode shapes. An improved set of basis vectors $X^{(1)}$ is now determined from

$$KX^{(1)} = Y^{(0)} \tag{12}$$

where

$$Y^{(0)} = M\Phi^{(0)} \tag{13}$$

The iterative process is continued until the eigenvalues determined from Eq. (10) converge to within a prescribed tolerance.

Subspace Iteration with Substructuring

In using the subspace iteration algorithm outlined in the preceding with substructuring, one needs to modify calculations of Eqs. (12) and (13) only. If one can perform these calculations with the substructural matrices rather than the system matrices, then one has a method of structural dynamics with substructuring. Calculations of Eqs. (12) and (13) can indeed be performed with only the substructural data. This is explained in the following.

Le us consider Eq. (13) first. Since the mass matrix M is generated by considering contribution to it from each finite element [10], Eq. (13) may be written as

$$Y^{(0)} = \left(\sum_{i=1}^{e} {}^{i}A^{T}\,{}^{i}m\,{}^{i}A \right) \Phi^{(0)} \tag{14}$$

where e is the number of finite elements, m^i is the mass matrix for the ith element and A^i is a Boolean transformation matrix taking contributions from m^i into M. Note that for each i, the matrix under the summation sign is an (n x n) matrix. Equation (14) may be rearranged as

$$Y^{(0)} = \sum_{i=1}^{e} \left\{ \left(A^{i^T} m^i A^i \right) \Phi^{(0)} \right\} \tag{15}$$

Thus calculations for $Y^{(0)}$ from Eq. (15) can be performed without ever generating the system mass matrix M. It is noted, however, that calculations of Eq. (13) must be performed in each subspace iteration. Therefore computational effort with Eq. (15) is greater as compared to calculations with a system mass matrix that is calculated once and stored in the computer core. This is due to the fact that calculations with Eq. (15) essentially imply generation of M at each subspace iteration. Thus, there is a trade-off between the computational effort and the computer core requirement for calculations of Eq. (13). Note that there is also a possibility of computing M only once and storing it on some external device such as a disk or magnetic tape. In every subspace iteration the stored M can be readily used. This procedure, however, will also require more computational time as compared to storage of M in the core because of the time delay in linking and rewinding the external device.

Calculations for $X^{(1)}$ from Eq. (12) can also be performed with only substructural matrices. The reason for this is that one can view Eq. (12) as an equilibrium equation governing static response of the system to a pseudo-load matrix $Y^{(0)}$ whose each column represents a

loading conditions. Equilibrium equations of the type of Eq. (12) can be solved using the substructuring concept [10]. The reader who is familiar with static structural analysis with substructures will readily recognize the following development. Others may consult Ref. 10 for more discussion on static structural analysis with substructures.

For the purpose of solving $X^{(1)}$ from Eq. (12) using the substructuring concept, one divides the entire structure into several substructures. Each substructure is viewed as a hyper-finite element that is connected to other substructures only at its interfaces. For each substructure a set of interior generalized coordinates and another set of interface or boundary generalized coordinates is defined. The hyper-element force-displacement relationships are derived in terms of only the boundary generalized coordinates by using essentially the static condensation procedure [10]. The hyper-element force-displacement relationships are synthesized to generate a reduced system of equilibrium equations in terms of only the boundary generalized coordinates for the system. This reduced system of equations is solved for boundary generalized coordinates and following the usual finite element approach, displacements at all points of the structure are calculated. This procedure is developed in the following.

First, let us partition Eq. (12) as follows:

$$\begin{bmatrix} K_{BB} & K_{BI} \\ \\ K_{IB} & K_{II} \end{bmatrix} \begin{bmatrix} X_B^{(1)} \\ \\ X_I^{(1)} \end{bmatrix} = \begin{bmatrix} Y_B^{(0)} \\ \\ Y_I^{(0)} \end{bmatrix} \tag{16}$$

where subscripts B and I refer to variables associated with the boundary and interior generalized coordinates for the entire structure. Now, using the second line of Eq. (16), $X_I^{(1)}$ is eliminated from the first line to obtain

$$K_B X_B^{(1)} = F_B \tag{17}$$

where

$$F_B = Y_B^{(0)} + Q^T Y_I^{(0)} \tag{18}$$

$$K_B = K_{BB} + K_{BI} Q \tag{19}$$

$$Q = - K_{II}^{-1} K_{IB} \tag{20}$$

It is interesting to note that matrices F_B and K_B in Eqs. (18) and (19) can be generated using the substructural matrices without ever generating K_{BB}, K_{II} or K_{BI} for the entire structure. For this purpose one also partitions the equilibrium equation for the rth substructures as

$$\begin{bmatrix} K_{BB}^r & K_{BI}^r \\ K_{IB}^r & K_{II}^r \end{bmatrix} \begin{bmatrix} X_B^{(1)r} \\ X_I^{(1)r} \end{bmatrix} = \begin{bmatrix} Y_B^{(0)r} \\ Y_I^{(0)r} \end{bmatrix} \tag{21}$$

where superscript r refers to the rth substructure. From the second line of Eq. (21), one obtains

$$K_{II}^{r} X_{I}^{(1)r} = Y_{I}^{(0)r} - K_{IB}^{r} X^{(1)r} \tag{22}$$

Eliminating $X_{I}^{(1)r}$ from the first line of Eq. (21), using Eq. (22), one obtains

$$K_{B}^{r} X_{B}^{(1)r} = F_{B}^{r} \tag{23}$$

where

$$K_{B}^{r} = K_{BB}^{r} + K_{BI}^{r} Q^{r} \tag{24}$$

$$F_{B}^{r} = Y_{B}^{(0)r} + Q^{r^{T}} Y_{I}^{(0)r} \tag{25}$$

$$Q^{r} = - [K_{II}^{r}]^{-1} K_{IB}^{r} \tag{26}$$

Equation (23) is a force-displacement relationship for the rth substructure in terms of its boundary generalized coordinate. The boundary stiffness matrix K_B and the effective boundary load matrix F_B for the entire structure are synthesized from the substructural data of Eqs. (24) and (25) using the standard finite element procedures:

$$K_{B} = \sum_{r=1}^{L} \beta^{r^{T}} K_{B}^{r} \beta^{r} \tag{27}$$

and

$$F_{B} = Y_{B}^{(0)} + \sum_{r=1}^{L} \beta^{r^{T}} Q^{r} Y_{I}^{(0)r} \tag{28}$$

where L is the number of substructures and β^r is a Boolean transformation matrix. It is noted here that the matrix K_{II}^r in Eq. (26) is usually not inverted. This matrix is symmetric and banded. Efficient procedures are used to decompose K_{II}^r and to solve for Q^r in Eq. (26) using forward and backward substitution. Also K_{IB}^r is usually quite sparse and advantage of this property is realized in calculations.

Now the matrix $X_B^{(1)}$ is calculated from Eq. (17). The matrices $X_I^{(1)r}$, r = 1, 2, ..., L, are calculated from Eq. (22). Finally the matrix $X^{(1)}$ is assembled from $X_B^{(1)}$ and $X_I^{(1)r}$. It is noted that the matrices, such as decompositions of K_B and K_{II}^r, Q^r, K_{IB}^r, and β^r are available from the static structural analysis for use in the preceding equations. Thus, the proposed subspace iteration algorithm with substructuring blends quite nicely into the static structural analysis with substructuring.

The <u>subspace iteration algorithm with substructures</u> is then summarized as follows:

<u>Step 1</u> Start with an (n x q) matrix $X^{(0)}$ whose columns are estimates of q eigenvectors.

<u>Step 2</u> Compute

$$Y^{(0)} = MX^{(0)} \tag{29}$$

using finite element operations similar to that of Eq. (15).

<u>Step 3</u> Solve for $\bar{X}^{(1)}$ from the equation

$$K\bar{X}^{(1)} = Y^{(0)} \tag{30}$$

using Eqs. (17) and (22).

Step 4 Compute

$$\bar{Y}^{(1)} = M\bar{X}^{(1)} \tag{31}$$

using finite element operations of Eq. (15).

Step 5 Calculate the following (q x q) matrices

$$K^* = \bar{X}^{(1)^T} Y^{(0)} \quad , \qquad M^* = \bar{X}^{(1)^T} Y^{(1)} \tag{32}$$

Step 6 Solve for all eigenvalues and eigenvectors of the projected eigenvalue problem of Eq. (10). Note that the generalized Jacobi iteration [14] or any other method of solving an eigenvalue problem may be used in this step.

Step 7 Compute

$$X^{(1)} = \bar{X}^{(1)} \Phi^* \quad , \qquad Y^{(1)} = \bar{Y}^{(1)} \Phi^* \tag{33}$$

Step 8 Check for convergence of eigenvalues. If all the ratios

$$\left| \omega_i^{2^{(1)}} - \omega_i^{2^{(0)}} \right| \Big/ \omega_i^{2^{(1)}} \tag{34}$$

are within a specified tolerance, then stop the iterative process. Otherwise return to Step 3 with $Y^{(0)} = Y^{(1)}$. After convergence, the first p columns of $X^{(1)}$ are the

required eigenvectors and the first p elements in Ω^{*2} are the corresponding eigenvalues.

The rate of convergence of the subspace iteration algorithm depends on how close the starting subspace is to the eigenvectors of the system. Many times eigenvectors from previous analyses are available and form an excellent starting subspace. For many structural problems, authors have simply used first q columns of an (n x n) identity matrix as the starting subspace [16-18]. The procedure suggested in Ref. 12 had also been used successfully [19,20]. Here the first column of $X^{(0)}$ contains simply diagonal elements of the mass matrix and other columns are unit vectors with +1 at the coordinates with largest ratios m_{ii}/k_{ii}.

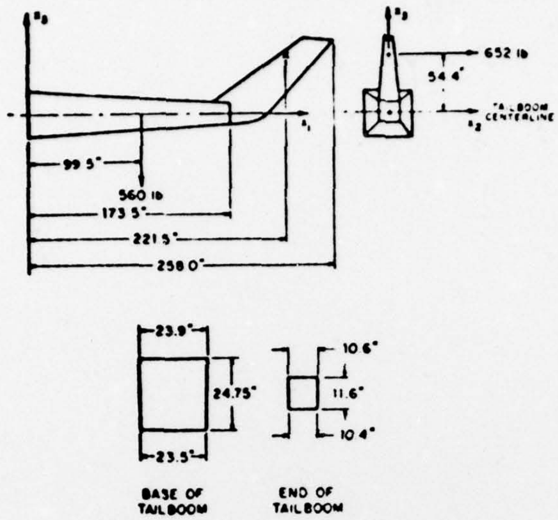
IV. AN EXAMPLE PROBLEM

As an example of calculating eigenvalues and eigenvectors with substructures, an open helicopter tail-boom structure is considered. The structure is modeled by 108 truss members and 28 joints as shown in Figure 1. The structure has 72 degrees of freedom, so the eigenvalue problem of Eq. (4) is of dimension 72 × 72. For sample calculations, cross-sectional area of all members is 1.0 in.^{2}, the material weight density is 0.10 lb/in.^{3} and Young's modulus is 10^6 psi.

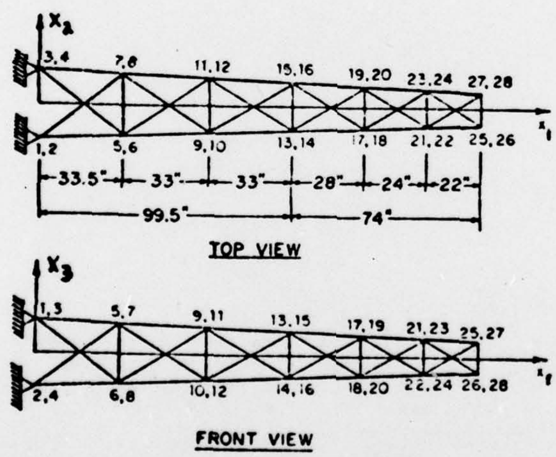
The first 10 natural frequencies and the corresponding mode shapes for the structure are obtained using the subspace iteration algorithms with and without substructuring. Two computer programs based on the subspace iteration have been developed, one without substructuring and the other with substructuring. These programs are used to obtain the results reported herein. Note that the generalized Jacobi iteration [14] is used to solve the eigenproblem of Eq. (10) in both programs.

For the algorithm without substructuring, the half-bandwidth of matrices K and M is 21. Thus 72 × 21 stiffness matrix is computed. It is decomposed and stored for use in Step 3 of the algorithm. The 72 × 21 mass matrix for the structure is also computed and stored for use in Steps 2 and 4 of the algorithm.

For the algorithm with substructuring, the tail-boom structure is divided into three substructures by partitioning it at nodes 9-12 and 17-20 as shown in Fig. 2. Nodes 25-28 are also treated as boundary nodes. Figure 2 shows global and local (or substructural) numbering systems for various substructures. Substructure 1 has 36 members, 12 boundary degrees

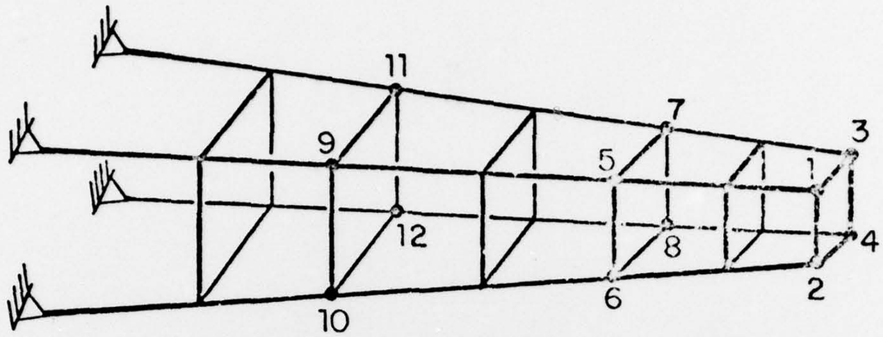


(a) Geometry of Helicopter Tail-Boom

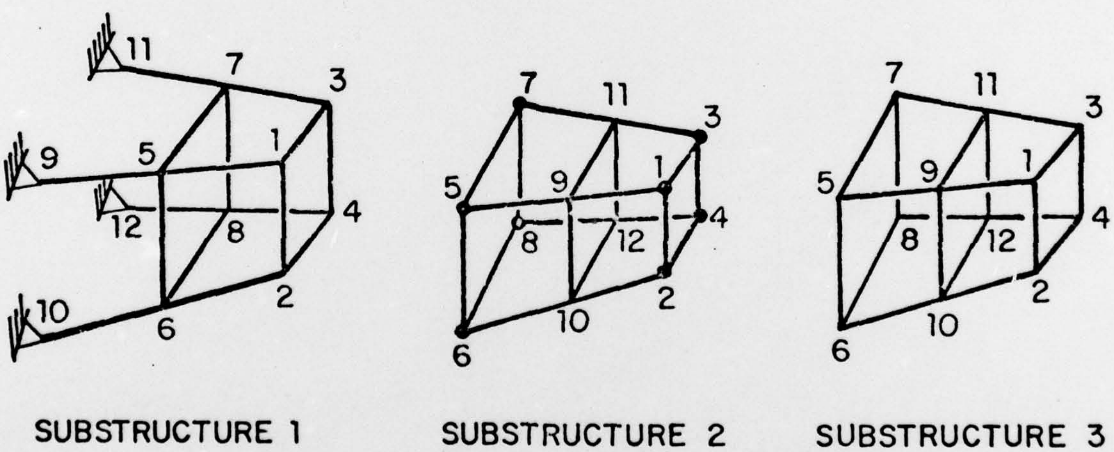


(b) **Finite Element Model for the Tail-Boom Structure (Degrees of freedom for joints 5 and 28 are 1, 2, 3 and 70, 71, 72, respectively.)**

Figure 1. Helicopter Tail-Boom Structure



(a) AN OVERALL NUMBERING SYSTEM FOR BOUNDARY NODES.



(b) NUMBERING SYSTEM FOR BOUNDARY AND INTERIOR NODES FOR EACH SUBSTRUCTURE.

NOTE: FOR CLARITY DIAGONAL MEMBERS ARE NOT SHOWN.

Figure 2. Nodal Numbering Systems for Substructural Formulation for the Finite Element Model of the Helicopter Tail-Boom Structure

of freedom and 24 interior degrees of freedom. Substructures 2 and 3 each have 24 boundary degrees of freedom, 12 interior degrees of freedom and 36 members. The boundary stiffness matrix K_B for the entire structure is a 36 × 36 matrix with half-bandwidth of 12. Decomposed part of this matrix is stored for use in Eq. (17). The decomposed part of the matrix K_{II}^r for each substructure is also stored for use in Eq. (26). The matrix Q^r is calculated from Eq. (26) and stored for each substructure. In this program the mass matrix for the structure is not stored. Rather, calculations of Eqs. (29) and (31) are carried out memberwise. Therefore, one should expect computational time to be slightly greater with this program.

The starting subspace for both programs is obtained by taking the first 18 columns of a 72 × 72 identity matrix. It should be noted, however, that degrees of freedom with substructuring formulation are numbered differently as compared to formulation without substructuring. Therefore, the starting subspace for the two programs is different. However, the two programs converged to exactly the same natural frequencies and the mode shapes. The results are summarized in the Table. The ten eigenvectors are listed in the Appendix. The tolerance limit for eigenvalue convergence in Step 8 was 10^{-6} for both programs.

For the results given in the table the subspace iteration algorithm with substructuring took 9 iterations to converge. The computing time was 72.3 sec on IBM 370-168(G) computer. The program based on the algorithm without substructuring took 13 iterations to converge with a computational time of 82.5 sec. The reason for four more iterations with this program is that the starting subspace is apparently not as good as

TABLE NATURAL FREQUENCIES WITH OR WITHOUT SUBSTRUCTURING

Natural Frequency No.	Natural Frequency in Hz
1	21.8236
2	23.1330
3	101.5989
4	105.2161
5	107.3932
6	200.8760
7	227.5787
8	239.3666
9	241.9844
10	377.4557

for the prévious program. However when the same starting subspace is chosen for the second program, the algorithm converges to the same solution in 9 iterations with a computational time of 61.0 sec. This time is less than the computing time with the substructuring program. The reason is that with the substructuring program, the mass matrix is calculated in each subspace iteration for use in Step 4 of the algorithm. When the substructuring program is modified to calculation of the mass matrix only once, the computing time is reduced to 57.8 sec.

V. DISCUSSIONS AND CONCLUSIONS

An efficient numerical procedure for structural dynamics with substructuring is developed. The method is based on the subspace iteration algorithm for calculating eigenvalues and eigenvectors of a general eigenproblem of Eq. (4). No approximating assumptions are made in developing the procedure. Thus, for a given finite element model of the structure, the natural frequencies and the corresponding mode shapes are the same with or without the substructuring method.

The method proposed herein does not use the component mode substitution ideas developed during the past 18 years. Thus, solution of the eigenproblem for each component is not necessary.

Conceptually the method is fairly straightforward and simple to program. Another advantage of the method is that it uses most of the data that is already available from structural analysis for static response. Thus the method blends quite nicely into static structural analysis with substructures. On the other hand this will not be true if component mode substitution methods are used, as some additional calculations with substructures would be necessary for obtaining static response of the structure.

It should be noted that the method of calculating natural frequencies and eigenvectors with substructures can be easily integrated into the optimal design algorithm with substructures [21] to treat constraints on natural frequencies of the structure. Also optimal design procedures with substructuring can be now developed for structures subjected to transient dynamic loads.

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EIGENVECTORS FOR OPEN TRUSS HELICOPTER TAIL-BOOM

AN APPENDIX

TO

DYNAMIC STRUCTURAL ANALYSIS WITH SUBSTRUCTURES

EIGENVECTOR NO. 1

0.9435D-02	0.1768D-01	-0.1903D-02	0.9271D-02	0.1768D-01
0.1898D-02	-0.9436D-02	0.1768D-01	0.1903D-02	-0.9271D-02
0.1768D-01	-0.1898D-02	0.1577D-01	0.6359D-01	-0.1183D-02
0.1549D-01	0.6360D-01	0.1171D-02	-0.1577D-01	0.6359D-01
0.1183D-02	-0.1549D-01	0.6360D-01	-0.1171D-02	0.1848D-01
0.1332D 00	-0.6813D-03	0.1815D-01	0.1332D 00	0.6594D-03
-0.1848D-01	0.1332D 00	0.6815D-03	-0.1815D-01	0.1332D 00
-0.6592D-03	0.1843D-01	0.2048D 00	-0.3265D-03	0.1810D-01
0.2048D 00	0.2993D-03	-0.1843D-01	0.2048D 00	0.3267D-03
-0.1810D-01	0.2048D 00	-0.2991D-03	0.1692D-01	0.2712D 00
-0.1170D-03	0.1661D-01	0.2712D 00	0.8748D-04	-0.1692D-01
0.2712D 00	0.1174D-03	-0.1661D-01	0.2712D 00	-0.8911D-04
0.1478D-01	0.3330D 00	-0.2991D-04	0.1450D-01	0.3331D 00
0.3730D-05	-0.1478D-01	0.3330D 00	0.3035D-04	-0.1450D-01
0.3331D 00	-0.3288D-05			

EIGENVECTOR NO. 2

-0.9924D-02	-0.1946D-02	0.1820D-01	0.9870D-02	0.1913D-02
0.1822D-01	-0.9924D-02	0.1946D-02	0.1820D-01	0.9870D-02
-0.1913D-02	0.1822D-01	-0.1657D-01	-0.1184D-02	0.6464D-01
0.1647D-01	0.1164D-02	0.6465D-01	-0.1657D-01	0.1184D-02
0.6464D-01	0.1647D-01	-0.1164D-02	0.6465D-01	-0.1946D-01
-0.6584D-03	0.1343D 00	0.1933D-01	0.6462D-03	0.1343D 00
-0.1946D-01	0.6585D-03	0.1343D 00	0.1933D-01	-0.6461D-03
0.1343D 00	-0.1950D-01	-0.3016D-03	0.2054D 00	0.1936D-01
0.2967D-03	0.2054D 00	-0.1950D-01	0.3017D-03	0.2054D 00
0.1936D-01	-0.2966D-03	0.2054D 00	-0.1806D-01	-0.9735D-04
0.2710D 00	0.1791D-01	0.9657D-04	0.2710D 00	-0.1806D-01
0.9748D-04	0.2710D 00	0.1791D-01	-0.9644D-04	0.2710D 00
-0.1598D-01	-0.1520D-04	0.3318D 00	0.1582D-01	0.1568D-04
0.3318D 00	-0.1598D-01	0.1536D-04	0.3318D 00	0.1582D-01
-0.1552D-04	0.3318D 00			

EIGENVECTOR NO. 3

-0.2283D-01	-0.1043D 00	0.2529D-02	-0.2237D-01	-0.1044D 00
-0.2510D-02	0.2284D-01	-0.1043D 00	-0.2566D-02	0.2237D-01
-0.1044D 00	0.2473D-02	-0.1067D-01	-0.2228D 00	-0.3485D-02
-0.1041D-01	-0.2231D 00	0.3578D-02	0.1067D-01	-0.2228D 00
0.3408D-02	0.1041D-01	-0.2231D 00	-0.3655D-02	0.2079D-01
-0.2345D 00	-0.5184D-02	0.2046D-01	-0.2349D 00	0.5386D-02
-0.2080D-01	-0.2345D 00	0.5105D-02	-0.2045D-01	-0.2349D 00
-0.5466D-02	0.4538D-01	-0.9901D-01	-0.3886D-02	0.4456D-01
-0.9935D-01	0.4134D-02	-0.4539D-01	-0.9901D-01	0.3854D-02
-0.4454D-01	-0.9935D-01	-0.4166D-02	0.5373D-01	0.1074D 00
-0.1604D-02	0.5274D-01	0.1072D 00	0.1879D-02	-0.5375D-01
0.1074D 00	0.1642D-02	-0.5272D-01	0.1072D 00	-0.1842D-02
0.4988D-01	0.3234D 00	-0.1886D-03	0.4893D-01	0.3233D 00
0.4766D-03	-0.4990D-01	0.3234D 00	0.2979D-03	-0.4892D-01
0.3233D 00	-0.3674D-03			

EIGENVECTOR NO. 4

0.5085D-03	0.5842D-01	0.5616D-01	-0.4524D-03	-0.5830D-01
0.5522D-01	-0.5026D-03	0.5842D-01	-0.5619D-01	0.4467D-03
-0.5830D-01	-0.5525D-01	0.4568D-03	0.1149D 00	0.1094D 00
-0.4020D-03	-0.1147D 00	0.1075D 00	-0.4542D-03	0.1149D 00
-0.1094D 00	0.3997D-03	-0.1147D 00	-0.1076D 00	0.3209D-03
0.1635D 00	0.1545D 00	-0.2976D-03	-0.1636D 00	0.1518D 00
-0.3265D-03	0.1635D 00	-0.1546D 00	0.3037D-03	-0.1636D 00

-0.1519D 00	0.1936D-03	0.1814D 00	0.1698D 00	-0.2005D-03
-0.1817D 00	0.1668D 00	-0.2056D-03	0.1814D 00	-0.1698D 00
0.2130D-03	-0.1817D 00	-0.1668D 00	0.7110D-04	0.1785D 00
0.1654D 00	-0.9226D-04	-0.1791D 00	0.1624D 00	-0.8523D-04
0.1785D 00	-0.1654D 00	0.1069D-03	-0.1791D 00	-0.1624D 00
0.1157D-04	0.1625D 00	0.1487D 00	-0.3461D-04	-0.1631D 00
0.1459D 00	-0.2478D-04	0.1625D 00	-0.1486D 00	0.4832D-04
-0.1631D 00	-0.1458D 00			

EIGENVECTOR NO. 5

-0.2295D-01	-0.2394D-02	0.1088D 00	0.2283D-01	0.2321D-02
0.1088D 00	-0.2294D-01	0.2338D-02	0.1088D 00	0.2284D-01
-0.2363D-02	0.1088D 00	-0.9581D-02	0.3602D-02	0.2267D 00
0.9524D-02	-0.3594D-02	0.2266D 00	-0.9576D-02	-0.3722D-02
0.2267D 00	0.9529D-02	0.3505D-02	0.2266D 00	0.2312D-01
0.5174D-02	0.2329D 00	-0.2298D-01	-0.5145D-02	0.2329D 00
0.2311D-01	-0.5305D-02	0.2330D 00	-0.2299D-01	0.5057D-02
0.2329D 00	0.4821D-01	0.3833D-02	0.9460D-01	-0.4789D-01
-0.3789D-02	0.9455D-01	0.4819D-01	-0.3903D-02	0.9463D-01
-0.4791D-01	0.3767D-02	0.9457D-01	0.5672D-01	0.1652D-02
-0.1094D 00	-0.5630D-01	-0.1576D-02	-0.1094D 00	0.5670D-01
-0.1625D-02	-0.1094D 00	-0.5632D-01	0.1650D-02	-0.1094D 00
0.5311D-01	0.3671D-03	-0.3196D 00	-0.5266D-01	-0.2217D-03
-0.3196D 00	0.5309D-01	-0.2374D-03	-0.3195D 00	-0.5268D-01
0.3940D-03	-0.3195D 00			

EIGENVECTOR NO. 6

-0.7824D-01	-0.9764D-02	0.9859D-02	-0.7819D-01	-0.9586D-02
-0.9683D-02	-0.7824D-01	0.9832D-02	0.9798D-02	-0.7819D-01
0.9526D-02	-0.9744D-02	-0.1479D 00	-0.6709D-02	0.7085D-02
-0.1478D 00	-0.6663D-02	-0.6560D-02	-0.1479D 00	0.6831D-02
0.6970D-02	-0.1478D 00	0.6543D-02	-0.6674D-02	-0.2004D 00
-0.4219D-02	0.4556D-02	-0.2005D 00	-0.4264D-02	-0.4205D-02
-0.2004D 00	0.4381D-02	0.4403D-02	-0.2005D 00	0.4100D-02
-0.4356D-02	-0.2327D 00	-0.2400D-02	0.2468D-02	-0.2328D 00
-0.2497D-02	-0.2595D-02	-0.2327D 00	0.2575D-02	0.2307D-02
-0.2328D 00	0.2327D-02	-0.2754D-02	-0.2495D 00	-0.9796D-03
0.8898D-03	-0.2495D 00	-0.1107D-02	-0.1276D-02	-0.2495D 00
0.1146D-02	0.7365D-03	-0.2495D 00	0.9423D-03	-0.1427D-02
-0.2551D 00	-0.1417D-03	0.5378D-04	-0.2551D 00	-0.2815D-03
-0.3834D-03	-0.2551D 00	0.2851D-03	-0.8247D-04	-0.2550D 00
0.1266D-03	-0.5170D-03			

EIGENVECTOR NO. 7

0.7581D-02	0.2199D 00	0.2629D-02	0.7306D-02	0.2205D 00
-0.2609D-02	-0.7568D-02	0.2199D 00	-0.2673D-02	-0.7319D-02
0.2205D 00	0.2565D-02	-0.3825D-01	0.2084D 00	0.5871D-02
-0.3755D-01	0.2092D 00	-0.6130D-02	0.3826D-01	0.2084D 00
-0.5974D-02	0.3754D-01	0.2092D 00	0.6028D-02	-0.3650D-01
-0.1055D 00	-0.3969D-02	-0.3570D-01	-0.1056D 00	0.3732D-02
0.3651D-01	-0.1055D 00	0.3816D-02	0.3569D-01	-0.1056D 00
-0.3884D-02	0.1905D-01	-0.2273D 00	-0.9473D-02	0.1876D-01
-0.2277D 00	0.9340D-02	-0.1905D-01	-0.2273D 00	0.9301D-02
-0.1876D-01	-0.2277D 00	-0.9512D-02	0.6275D-01	-0.3162D-01
-0.6136D-02	0.6155D-01	-0.3171D-01	0.6028D-02	-0.6276D-01
-0.3162D-01	0.5971D-02	-0.6155D-01	-0.3171D-01	-0.6194D-02
0.6855D-01	0.2866D 00	-0.1493D-02	0.6717D-01	0.2868D 00
0.1312D-02	-0.6855D-01	0.2866D 00	0.1343D-02	-0.6716D-01
0.2868D 00	-0.1462D-02			

EIGENVECTOR NO. 8

0.5489D-02	-0.3208D-02	-0.2261D 00	-0.5516D-02	0.2930D-02
-0.2260D 00	0.5495D-02	0.2760D-02	-0.2257D 00	-0.5516D-02
-0.2507D-02	-0.2256D 00	-0.4071D-01	-0.6029D-02	-0.2036D 00
0.4035D-01	0.5724D-02	-0.2034D 00	-0.4071D-01	0.5359D-02
-0.2030D 00	0.4035D-01	-0.5107D-02	-0.2028D 00	-0.3583D-01
0.3975D-02	0.1145D 00	0.3538D-01	-0.3855D-02	0.1144D 00
-0.3584D-01	-0.4480D-02	0.1150D 00	0.3538D-01	0.4304D-02
0.1149D 00	0.2206D-01	0.9156D-02	0.2246D 00	-0.2223D-01
-0.8867D-02	0.2244D 00	0.2205D-01	-0.9348D-02	0.2248D 00
-0.2223D-01	0.9036D-02	0.2246D 00	0.6602D-01	0.5756D-02
0.2512D-01	-0.6588D-01	-0.5614D-02	0.2504D-01	0.6602D-01
-0.5722D-02	0.2510D-01	-0.6589D-01	0.5606D-02	0.2502D-01
0.7198D-01	0.1328D-02	-0.2834D 00	-0.7172D-01	-0.1309D-02
-0.2834D 00	0.7198D-01	-0.1198D-02	-0.2835D 00	-0.7173D-01
0.1258D-02	-0.2835D 00			

EIGENVECTOR NO. 9

-0.1044D-02	-0.1488D 00	-0.1420D 00	0.9163D-03	0.1486D 00
-0.1395D 00	0.9040D-03	-0.1488D 00	0.1426D 00	-0.1011D-02
0.1486D 00	0.1400D 00	0.1613D-03	-0.1940D 00	-0.1835D 00
-0.2744D-03	0.1941D 00	-0.1801D 00	-0.2997D-03	-0.1940D 00
0.1840D 00	-0.3101D-04	0.1941D 00	0.1807D 00	0.1039D-02
-0.7368D-01	-0.6909D-01	-0.1166D-02	0.7378D-01	-0.6783D-01
-0.1243D-02	-0.7366D-01	0.6896D-01	0.7682D-03	0.7379D-01
0.6769D-01	0.9645D-03	0.7870D-01	0.7331D-01	-0.1153D-02
-0.7891D-01	0.7196D-01	-0.1317D-02	0.7872D-01	-0.7379D-01
0.8058D-03	-0.7892D-01	-0.7245D-01	0.3824D-03	0.1689D 00
0.1558D 00	-0.6266D-03	-0.1693D 00	0.1529D 00	-0.8439D-03
0.1689D 00	-0.1561D 00	0.3379D-03	-0.1693D 00	-0.1532D 00
0.1378D-04	0.1863D 00	0.1700D 00	-0.2943D-03	-0.1866D 00
0.1667D 00	-0.4992D-03	0.1863D 00	-0.1698D 00	0.1256D-04
-0.1866D 00	-0.1665D 00			

EIGENVECTOR NO. 10

0.3309D-03	0.1769D 00	0.1686D 00	-0.2074D-03	-0.1787D 00
0.1653D 00	-0.3343D-03	0.1769D 00	-0.1686D 00	0.2144D-03
-0.1787D 00	-0.1653D 00	-0.1924D-02	0.4288D-01	0.3995D-01
0.1988D-02	-0.4233D-01	0.3909D-01	0.1927D-02	0.4288D-01
-0.3990D-01	-0.1986D-02	-0.4233D-01	-0.3905D-01	-0.4473D-03
-0.2020D 00	-0.1893D 00	0.3379D-04	0.2044D 00	-0.1856D 00
0.4541D-03	-0.2020D 00	0.1893D 00	-0.3841D-04	0.2044D 00
0.1856D 00	0.1811D-02	-0.8731D-01	-0.7950D-01	-0.1794D-02
0.8544D-01	-0.7785D-01	-0.1815D-02	-0.8730D-01	0.7947D-01
0.1794D-02	0.8544D-01	0.7782D-01	0.1843D-02	0.1159D 00
0.1073D 00	-0.9941D-03	-0.1178D 00	0.1053D 00	-0.1857D-02
0.1159D 00	-0.1073D 00	0.9976D-03	-0.1178D 00	-0.1053D 00
0.1175D-02	0.2010D 00	0.1812D 00	-0.1231D-03	-0.1985D 00
0.1777D 00	-0.1192D-02	0.2010D 00	-0.1812D 00	0.1263D-03
-0.1985D 00	-0.1777D 00			